MATH 918: TOPICS IN COMMUTATIVE ALGEBRA
SPRING 2026: F-SINGULARITIES

INSTRUCTOR: ANNA BROSOWSKY

Currently scheduled for Tuesday/Thursday, 11am-12:15pm. Time subject to change, check registrar for most up-to-date info.

1. Description:

The "F" in F-singularities refers to a powerful tool in prime characteristic commutative algebra: If $p > 0$ is a prime number and ring R has characteristic p, then the *Frobenius map* is the ring homomorphism $r \mapsto r^p$. The first bit of magic is that this is a ring map at all! We need the *freshman's dream*, which says $(r + s)^p = r^p + s^p$. Beyond this, the Frobenius can be used to detect how "good" or "bad" our ring is, such as whether R is *reduced* or even better whether R is *regular*. This leads us to the full name "F-singularities": The study/classification of singularities (i.e., descriptions of how "nice" a ring is) done using the Frobenius.

The goal of this course is to give an overview of the key objects and questions of interest that appear when working in this area. A tentative list of topics is below. If you've heard any of these words appear in seminar talks, come take this class to learn what they mean!

- Basics of working in characteristic p, including char p fields and restriction of scalars along the Frobenius.
- Singularities defined in terms of having splittings of (many) R-module maps related to the Frobenius, namely, Frobenius splitting and strong F-regularity. Including consequences of and (some) ways to test for this.
- Singularities defined in terms of the Frobenius action on local cohomology, namely, F-rationality and F-injectivity.
- Closure operations: tight closure & Frobenius closure, and how they relate to the above.
- An introduction to the technique of "reduction to characteristic p".
- Connections to other areas of commutative algebra! Will depend on interest, but could include homological algebra, differential operators, and/or big Cohen-Macaulay algebras.
- Numerical invariants: some finer-grained ways to understand how "good" or "bad" a ring is, such as the Hilbert-Kunz multiplicity, the F-pure threshold, and the F-signature.

2. Prerequisites:

The pre/co-requisites are the commutative algebra sequence: Math 905 (Commutative Algebra I), Math 953 (Algebraic Geometry), Math 915 (Homological Algebra), and being currently enrolled in or having already taken Math 906 (Commutative Algebra II). That said, if you are interested but are missing some of the pre-reqs, send me an email and we can discuss it!

Date: 11/15/2025.

3. References:

The main references used during this course will likely be both

- Linquan Ma & Thomas Polstra's F-Singularities: A Commutative Algebra Approach
- Alessio Caminata & Alessandro de Stefani's Notes for course on F-singularities

4. Coursework and grading:

The course grade will be based on regular problem sets.